NOTE ON THE DIVISION OF A PLANE BY A POINT SET*

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A plane set of points K is said to divide a plane S if the set $S - K$ is composed of two mutually exclusive domains S_1, S_2, of which K is a common boundary, where by domain is meant a connected open set. The condition that K be a simple closed curve or an open curve has been stated by J. R. Kline† in terms of the concept "connected im kleinen." In proving that the set K is a connected set, Kline employs the condition "connected im kleinen."

If we assume that K is bounded and that we have at our disposal the parallel and perpendicular straight lines of a number plane, then the connectedness of K is established by other writers, for example Hausdorff, *Grundzüge der Mengenlehre,* page 346, Theorem XII. Hausdorff calls attention, in a footnote to page 342, to the difficulty of extending his argument to the case of unbounded sets.

It seems in view of the importance of the theory of open curves as indicated by R. L. Moore‡ and of the importance in general of the fundamental theorems of plane analysis situs that it is of interest to show that the set K is connected, whether bounded or not, without the use of the restriction employed by Kline or of the properties of straight lines and rectangles. The present note is concerned, therefore, with the proof of the following theorem.

THEOREM. *Let K be a plane point set, S, the set of all points of the plane, and denote by S_1, S_2 two mutually exclusive domains such that*

$$S - K = S_1 + S_2.$$

Then if every point of K is a limit point of both S_1 and S_2, the set K is closed and connected.

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† *Concerning approachability of simple closed and open curves.* TRANSACTIONS OF THIS SOCIETY, vol. 21 (1920), pp. 451–458.

‡ R. L. Moore, *On the foundations of plane analysis situs.* TRANSACTIONS OF THIS SOCIETY, vol. 17 (1916), pp. 131–164. This paper will be referred to as "Foundations."