

## SHORTER NOTICES

*Oeuvres de G. H. Halphen.* Publiées par les soins de C. Jordan, H. Poincaré, É. Picard, avec la collaboration de E. Vessiot. Tome III.\* Paris, Gauthier-Villars, 1921. xii + 518 pp.

The Paris Academy of Sciences proposed as the subject of the grand prize in mathematics for 1880 the following question: "To perfect in some important part the theory of ordinary linear differential equations." Halphen submitted a memoir in which he proposed to determine those ordinary linear differential equations which are reducible to integrable forms by means of transformations of a certain class; and the prize was awarded to this memoir. The text of Hermite's report on the memoir is reproduced in an article on Halphen published by C. Jordan in LIOUVILLE'S JOURNAL in 1889 and reprinted on pages v to xii of the volume under review. (See also p. 33 of vol. I of Halphen's *Oeuvres*.)

Halphen formulated in the following manner the exact "double question" which he treated:

(1) Having given a linear differential equation with variable  $X$  and unknown  $Y$ , to determine if there exists a substitution

$$x = \varphi(X), \quad y = Y\psi(X),$$

such that,  $x$  and  $y$  being taken for the new variable and the new unknown, the transformed equation belongs to one of the following three categories:

- I. Equations with constant coefficients;
- II. Equations whose general integral is rational;
- III. Equations with doubly periodic coefficients with the same periods and with a uniform general integral.

(2) Having determined the existence of the substitution, to effect the integration.

The memoir in which he treats this question is reprinted on pages 1-260 of the volume under review. It opens with an excellent fourteen-page summary of the results attained. The response to the first question is entirely decisive: necessary and sufficient conditions are obtained (and summarized in the introduction) for the existence of the substitution named of such sort that one may apply them to a particular equation by means of straightforward reckoning. The propositions relating to the second part are not quite so definitive; but they are far-reaching enough to have important applications, particularly to equations of the second order.

The judgment of the Academy, awarding the grand prize to Halphen for this memoir, was scarcely rendered when the Berlin Academy proposed for the Steiner prize "the solution of an important question in the theory of twisted algebraic curves." In 1870 Halphen had published a short note on the classification of these curves. He now returned to this subject and produced his classic memoir on the classification of twisted algebraic curves (reprinted on pp. 261-455 of the volume under review). It was crowned conjointly with a memoir by Noether submitted on the same

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\* For a short review of volumes I and II see this BULLETIN, vol. 27 (1920-21), pp. 466-468.