

Géométrie Synthétique des Unicursales de Troisième Classe et de Quatrième Ordre. By E. Bally. Paris, Gauthier-Villars, 1920. 98 pp.

As its title indicates, this book develops the properties of unicursal curves of the third class and the fourth order synthetically. These curves are homographically equivalent to the tri-cuspidal hypocycloid or to the cardioid. The discussion is therefore limited to these cycloids. The ordinary cycloid is included as a limiting case.

If E and I are two concentric circles of radii R_e and R_i , respectively, ($R_e > R_i$), there are two families of circles tangent to both whose diameters are $R_e + R_i$ and $R_e - R_i$. Consider two circles, one from each family, meeting in a point A and touching E at E_1 and E_2 . If these circles roll upon E so that E_1 and E_2 move in opposite directions with velocities proportional to the radii of the rolling circles, A remains fixed in position upon each circle and describes a hypocycloid. If the circles roll upon I so that their points of contact move in the same direction with velocities proportional to the radii, A will describe an epicycloid.

The properties of the tri-cuspidal hypocycloid and of the cardioid are developed in Chapter I, starting from the foregoing mode of generation.

Chapter II deals with the tangential properties of these curves as correlative to point properties of nodal cubics.

The two following chapters deal with properties deducible from the fact that a tri-cuspidal quartic is the transform, by quadric transformation, of a conic inscribed in the triangle of its cusps.

Chapter V deals with the nodal cubic as a projection of a twisted cubic, and the tri-cuspidal quartic as a section of the developable surface formed by the tangents to the twisted cubic.

The book is interesting in that it develops the properties of these special curves in a simple way and by elementary means, but its value would be greatly enhanced to the interested reader had full references to original sources been included. For example, no reference is made to the papers of Steiner and Cremona (CRELLE, vols. 53 and 64, respectively) upon these curves. Perhaps such references will be given in the larger work on geometry which is promised in the preface, but which has not yet appeared.

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Éléments de Géométrie, I et II, by Alexis-Claude Clairaut. (Les Maîtres de la Pensée Scientifique.) Paris, Gauthier-Villars, 1920. 14 + 96 + 104 pp.

This is an opportune time for the re-publication of Clairaut's *Éléments de Géométrie* that was first brought out at Paris in 1741 and again in 1753. The present-day tendencies in the teaching of beginners' geometry are in some way those set forth by Clairaut 180 years ago. Many teachers will be glad to examine this eighteenth century rival of Euclid's *Elements*, a text written by one of the most distinguished French mathematicians of that century. At first Clairaut brings into play chiefly the intuitive powers. He secures the interest of beginners by frequent reference to exercises in surveying. The reprint appears in two small volumes.

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