tain exactly 7 subgroups of order 9. Its order would therefore be 63, 126, or 252. This is impossible as each of these groups would involve only one subgroup of order 7 since each of its subgroups of order 7 would be transformed into itself by at least 21 substitutions.

It remains only to consider the case when G_1 would contain a substitution of order 3 and of degree 60 without involving such a substitution of degree 63. The order of the group formed by all the substitutions of G which would be commutative with this substitution of order 3 would be 90. This group of order 90 would transform its ten subgroups of order 9 according to a transitive group of order 30 and of degree 10. Since this transitive group does not exist,* we have arrived at nothing but contradictions by assuming the existence of a second simple group of order $7!/2$ and hence such a group is actually proved to be non-existent.

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A THEOREM OF OSCILLATION

BY W. E. MILNE

In an investigation of the oscillations of aerial bombs a need was found for the following proposition. Both the theorem and its proof are modelled after a similar theorem and proof by Osgood.†

THEOREM. *Let $\varphi(t)$ be positive, continuous, monotonically increasing, and bounded in the interval $T \leqq t < \infty$, and let m and M be two positive constants such that $m < \varphi(t) < M$ for $t > T$. Let $f(y)$ be an odd, monotonically increasing function, satisfying the Lipschitz condition*

$$|f(y_1) - f(y_2)| < K|y_1 - y_2|, \qquad K > 0,$$

in an interval $-a \leqq y \leqq +a$, $a > 0$. Let y be a solution of the differential equation

$$\frac{d^2y}{dt^2} + \varphi(t)f(y) = 0 \tag{1}$$

* Cf. F. N. Cole, QUARTERLY JOURNAL, vol. 27 (1895), p. 40.

† This BULLETIN, vol. 25 (1919), pp. 216–221.