

DE LA VALLÉE POUSSIN ON APPROXIMATIONS

Leçons sur l'Approximation des Fonctions d'une Variable Réelle. By C. de la Vallée Poussin. Paris, Gauthier-Villars, 1919. vi + 150 pp.

In 1908, M. de la Vallée Poussin was engaged in a study of certain questions in the theory of approximate representation by means of polynomials and finite trigonometric sums. In discussing the representation of a function whose graph is a broken line, he was led by the limitations of the formulas which he employed to make the following observation:*

"Il serait très intéressant de savoir s'il est impossible de représenter l'ordonnée d'une ligne polygonale avec une approximation d'ordre supérieur à $1 : n$ par un polynôme de degré n ."

This sentence, occurring incidentally, with no particular emphasis, in a footnote attached to some supplementary pages following a memoir on a different phase of the subject, has been the direct or indirect occasion of about thirty published articles and memoirs which I call to mind at the moment,† and probably of numerous others which would be disclosed by a thorough search of the literature. The resulting theory is concerned not only with inner limits of approximation, as contemplated in the passage just quoted, but also with outer limits for the approximation attainable by various means, the degree of convergence of Fourier's and other series, and a variety of related topics. While this theory also has other origins and beginnings,‡ it is a fact that the numerous papers referred to can in each case be traced back in the personal experience and associations of the authors at least partly to de la Vallée Poussin's formulation of the problem.

The book under review, a monograph in the Borel series, is a summary, not of the entire literature of the subject, for citations are few and informal, but of its principal results, in systematic and often novel presentation. It is particularly appropriate that the man to whom the theory chiefly owes its inception, a man who has made essential contributions to it at various stages of its development, should now have performed the service of setting it before the general reader in its most attractive form.

Of the contents of the book in detail I shall speak more briefly than would otherwise be desirable, for the reason that I had occasion to refer to it extensively, as well as to the other literature, in an expository paper recently published in this BULLETIN.§

* de la Vallée Poussin, *Note sur l'approximation par un polynôme d'une fonction dont la dérivée est à variation bornée*, BULLETINS DE L'ACADÉMIE DE BELGIQUE, CLASSE DES SCIENCES, 1908, pp. 403-410; p. 403, footnote.

† It will be a convenience to the reviewer in the present connection if he may be allowed the occasional use of the pronoun in the first person.

‡ See particularly Lebesgue, *Sur la représentation approchée des fonctions*, RENDICONTI DI PALERMO, vol. 26 (1908), pp. 325-328.

§ D. Jackson, *The general theory of approximation by polynomials and trigonometric sums*. *Chicago Symposium paper*. March 25, 1921. This BULLETIN, vol. 27, Nos. 9-10, June-July, 1921, pp. 415-431.