ON KAKEYA'S MINIMUM AREA PROBLEM*

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1. *Introduction.* During recent years the Japanese school of mathematicians, notably Professors Hayashi, Kakeya and Fujiwara, have proposed and investigated to some extent a unique and apparently new class of maxima-minima problems of which the one considered in this paper may be regarded as the simplest type. In general, such problems concern the determination of the closed curve of least area within which a given configuration may be completely rotated. The special problem in which we shall be interested appears to have been first stated by Kakeya and is as follows:†

A line-segment AB lying in the plane MN is to be moved so that it shall return to its original position but with its ends reversed (as in the rotation of a segment about its middle point through a semicircumference). How should this be done in order that the area generated during the motion may be a minimum?

2. *Interpretations of the Problem.* As thus stated, we note first that the problem admits of the following two interpretations: In computing area generated during any portion of the motion, the area S bounded by any given enclosure in the plane MN is to be counted (a) as many times as it is passed over by AB; (b) never more than once.

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† The existing literature upon this and the more general problems above referred to appears to be chiefly confined to the following three papers: *On the curves of constant breadth, and the convex closed curves inscribable and revolvable in a regular polygon*, by Tsuruichi Hayashi, TÔHOKU SCIENCE REPORTS, vol. 5, pp. 303–312 (Dec. 1916); *On some problems of maxima and minima for the curve of constant breadth and the in-revolvable curve of the equilateral triangle*, by M. Fujiwara and S. Kakeya, TÔHOKU JOURNAL, vol. 11, pp. 92–110 (Feb. 1917); *Some problems on maxima and minima regarding ovals*, by Soichi Kakeya, TÔHOKU SCIENCE REPORTS, vol. 6, pp. 71–88 (July, 1917). Recently Pál (MATHEMATISCHE ANNALEN, vol. 83 (1921), pp. 311–319) has given a complete solution, but under greater restrictions than those of this paper.