REPORT ON
TOPICS IN THE THEORY OF DIVERGENT SERIES*

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1. *Introduction.* It is with some reluctance that I have acceded to the request of the programme committee to address the Society on the present status of problems concerning divergent series, since three admirable expository treatments of this field have already been given to the Society and published in the BULLETIN, by W. B. Ford,† R. D. Carmichael,‡ and C. N. Moore.§ In particular Professor Carmichael has so closely followed the trend which my own thoughts have taken (except that I should not have presented them so elegantly) as to make it difficult for me to give an adequate account without a good deal of repetition. At any rate I shall take advantage of his paper in order to plunge *in medias res* today, and also to omit references to original sources unless these seem especially desirable.

If we have the symbol

$$\Sigma u_n = u_1 + u_2 + u_3 + \cdots, \tag{1}$$

we define

$$x_n = u_1 + u_2 + \cdots + u_n, \tag{2}$$

so that

$$u_1 = x_1; \qquad u_n = x_n - x_{n-1}, \quad n > 1. \tag{3}$$

In case

$$\lim_{n \to \infty} x_n$$

exists, we say that the series Σu_n or the sequence (x_n) is convergent, the limit of the sequence being the *value* or *sum* of the series. The importane of this concception lies in the fact that many formal transformations carried out on infinite series as if they are finite sums can be proved correct. Instances arose early in the study of series, however, in which

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† This BULLETIN, vol. 25 (1918–19), p. 1.

‡ *Ibid.*, vol. 25 (1918–19), p. 97.

§ *Ibid.*, vol. 25 (1918–19), p. 258.