

THE GENERAL THEORY OF APPROXIMATION BY POLYNOMIALS AND TRIGONOMETRIC SUMS.

*REPORT PRESENTED BEFORE THE AMERICAN MATHEMATICAL
SOCIETY AT THE SYMPOSIUM IN CHICAGO ON
MARCH 25, 1921.*

BY PROFESSOR DUNHAM JACKSON.

1. *Introduction.* A great part of the progress made in mathematical analysis during the last hundred years has been closely related, either logically or historically, to the study of Taylor's and Fourier's series. The power series is fundamental, or can be made fundamental, for the theory of functions of complex variables, and the theory of functions of real variables has been elevated to its present dignity and scope largely by the successive additions made to meet the demands of the inquirer into the properties of trigonometric series.*

One of the outlying portions of the structure is built around the problem of the approximate representation of an arbitrary function by means of polynomials or by means of finite trigonometric sums in general; that is, with the admission of coefficients other than those of a specified number of terms of a particular series. This theory, which has grown to its present extent mainly within twenty years, forms the subject of the following report. It can not be set off from the rest by any sharp line of demarcation, but there are certain well-marked processes of development which it is not difficult to trace. It has seemed expedient for various reasons to make this paper an introduction to the literature of the subject, rather than an independent exposition of any considerable part of the theory. Even with this limitation, the treatment is merely illustrative, not in any sense exhaustive.

2. *Tchebychef's Theory and its Generalizations.* Between 1850 and 1860, Tchebychef † discussed the problem of de-

* Cf. Van Vleck, *The influence of Fourier's series upon the development of mathematics*, SCIENCE, vol. 39 (1914), pp. 113-124.

† Tchebychef 1 (this form of reference will be adopted when there is occasion to cite more than one paper by the same author), *Théorie des mécanismes connus sous le nom de parallélogrammes*, MÉMOIRES PRÉSENTÉS À L'ACADÉMIE IMPÉRIALE DES SCIENCES DE ST.-PÉTERSBOURG PAR DIVERS SAVANTS, vol. 7 (1854), pp. 539-568; *Oeuvres*, vol. 1, Petrograd, 1899, pp. 111-143.

Tchebychef 2, *Sur les questions de minima qui se rattachent à la représenta-*