THE TRANSFORMATION OF ELLIPTIC INTEGRALS.

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1. *Introduction.* Jacobi discovered the transformation of the fifth order and proposed the problem of the transformation of order n. The solution of a system of algebraic equations is required. The number of arbitrary quantities is equal to the number of equations, but the direct solution could not be effected. The introduction of the inverse functions and the periods furnished a transcendental solution which cannot be regarded as complete till the transcendental elements are eliminated since the periods are not given. Cayley attempted an algebraic solution without success, as did also Clifford. Cayley says:

"The extension of this algebraic theory (Jacobi's determination of the transformations of degrees 3 and 5) to any value whatever of n is a problem of great interest and difficulty: such theory should admit of being treated in a purely algebraical manner; but the difficulties are so great that it was found necessary to discuss it by means of the formulae of the transcendental theory, in particular by means of the expressions involving Jacobi's q (the exponential of $-\pi k'/k$), $\cdots$. In the present memoir I carry on the theory algebraically as far as I am able; and I have, it appears to me, put the purely algebraical question in a clearer light than has hitherto been done; but I still find it necessary to resort to the transcendental theory."

In what follows the solution of Jacobi's system of equations is given independently of the transcendental theory and the foundation laid for a purely algebraic treatment of the whole subject of transformations.

2. *Jacobi's Problem.* Let s, Σ be two conics such that it is possible to find a polygon of n sides inscribed in s and circumscribed about Σ. Then, by a well known theorem, there can be described an infinity of polygons having the same property. Let a parameter t be introduced on the conic s; then the values of the parameters of the vertices of any polygon of the system are given by an equation $f(t, \lambda) = 0$ of degree n in t and of the first degree in λ.