EXTENSION OF AN EXISTENCE THEOREM FOR A NON-SELF-ADJOINT LINEAR SYSTEM.

BY PROFESSOR H. J. ETTLINGER.

(Read before the American Mathematical Society December 31, 1919.)

In a recent paper* the writer established the existence of at least one real characteristic number for the non-self-adjoint system

$$(1) \qquad \frac{d}{dx}\left[K(x, \lambda) \frac{du}{dx} \right] - G(x, \lambda)u = 0,$$

$$(2) \quad U_i = A_{i1}u(a) - A_{i2}K(a)u_x(a) - A_{i3}u(b) + A_{i4}K(b)u_x(b) = 0 \quad (i = 1, 2),$$

satisfying the following conditions I, II, III, IV, V, and either VI A or VI B:

I. $K(x, \lambda)$ and $G(x, \lambda)$ are continuous, real functions of x in $a \leqq x \leqq b$ and for all real values of λ in the interval

$$\Lambda(\Lambda_1 < \lambda < \Lambda_2).$$

II. $K(x, \lambda)$ is positive everywhere in (a, b), Λ.

III. The sets of real constants A_{1j} and A_{2j} are not proportional.

IV. For each value of x in (a, b), K and G do not increase as λ increases.

V. $\lim_{\lambda=\Lambda_1} - \frac{\min G}{\min K} = -\infty, \quad \lim_{\lambda=\Lambda_2} - \frac{\max G}{\max K} = +\infty.$

VI A.† $D_{12} \cdot D_{34} = 0$, together with either

$$(a) \qquad D_{24}^2 + D_{14}^2 \neq 0, \quad D_{14} \geqq 0, \quad D_{24} \geqq 0,$$

or

$$(b) \quad D_{24}^2 + D_{14}^2 = 0, \quad D_{13}^2 + D_{23}^2 \neq 0, \quad D_{13} \leqq 0, \quad D_{23} \leqq 0.$$

VI B. $D_{12} \cdot D_{34} \neq 0$, together with either

$$(a)\ D_{24} > 0 \quad \text{or} \quad (b)\ D_{24} = 0, \quad D_{14} > 0.$$

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* *Existence theorem for the non-self-adjoint linear system of the second order*, ANNALS OF MATHEMATICS, vol. 25 (1920), pp. 278–290.

† $D_{ij} = \begin{vmatrix} A_{1i} & A_{1j} \\ A_{2i} & A_{2j} \end{vmatrix}.$