4. Write $u_k(m) = m^k$, and put

$$X_k'(m) = X_k(m)/m^k, \qquad Z_k'(m) = Z_k(m)/m^k.$$

Then from the definitions of the functions,

$$X_\mu' = u_{-\mu}f, \qquad Z_\mu' = u_{-\mu}F,$$

and from the associative and commutative laws,

$$u_{-\mu}F \cdot u_{-\nu}f = u_{-\mu}f \cdot u_{-\nu}F,$$

we find $Z_\mu' X_\nu' = X_\mu' Z_\nu'$, which may be written in full as follows:

$$\Sigma_m \frac{Z_\mu(\delta)}{\delta^\mu} \frac{X_\nu(d)}{d^\nu} = \Sigma_m \frac{X_\mu(\delta)}{\delta^\mu} \frac{Z_\nu(d)}{d^\nu}.$$

Multiplying this throughout by m^μ, we get (A).

THE UNIVERSITY OF WASHINGTON,
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A SEQUENCE OF POLYNOMIALS CONNECTED WITH THE nTH ROOTS OF UNITY.

BY DR. T. H. GRONWALL.

(Read before the American Mathematical Society September 7, 1920.)

In constructing examples of power series bounded in their circle of convergence and having specified convergence defects on the circle, it is frequently useful to consider polynomials of degree $n - 1$, such that at each of the nth roots of unity, the absolute value of the polynomial is less than or equal to a given constant M. Under these conditions, the maximum absolute value of the polynomial inside or on the unit circle is less than $4M \log n$.*

It is the purpose of this note to determine those polynomials where this maximum is as large as possible. The result may be stated in the following theorem.

THEOREM. *When the polynomial*

$$F(z) = a_0 + a_1 z + \cdots + a_{n-1} z^{n-1}$$

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* E. Landau, *Bemerkungen zu einer Arbeit des Herrn Carleman*, MATHEMATISCHE ZEITSCHRIFT, vol. 5 (1919), pp. 147–153.