where the last reduction was made by interchanging integrations in (23) and by comparing the result with (22). The interchange was permissible* because of our present hypotheses. Since $[z_i(x') - z_i(x)]$ is the ith Fourier coefficient of the left side of (20), we have completed the proof of the theorem.

In a later paper the author will consider applications of the present results in the theory of functionals whose arguments are summable functions.

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NOTE ON MINIMAL VARIETIES IN HYPERSPACE.

BY PROFESSOR C. L. E. MOORE.

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1. It is known that a necessary and sufficient condition that a surface of two dimensions in hyperspace be minimal is the vanishing of the vector mean curvature.† It is the purpose of this note to show that mean curvature of a variety V_m in a space of n dimensions can be defined in the same way and that its vanishing is a necessary and sufficient condition that V_m be minimal. I shall use the absolute calculus, since one of its chief merits is the ease with which invariants can be written down. In fact the very form of an expression shows whether or not it is invariant. Enough vector analysis is used to simplify the form of the expressions.

The variety V_m can be written vectorially in the form

$$y = y(x_1, x_2, \cdots, x_m).$$

Then

$$ds^2 = dy \cdot dy = \sum_1^m a_{rs} dx_r dx_s.$$

If we write

$$y_r = \frac{\partial y}{\partial x_r},$$

* Cf. de la Vallée Poussin, loc. cit., p. 53.

† Wilson and Moore, "Differential geometry of two-dimensional surfaces in hyperspace," *Proceedings of the Amer. Acad.*, vol. 52 (1916).