$$\begin{array}{lll} & \text{trajectories,} & \text{if } F = \sqrt{W_2 + h}, \\ (18) & \text{brachistochrones,} & \text{if } F = 1/\sqrt{W_2 + h}, \\ & \text{catenaries,} & \text{if } F = W_2 + h. \end{array}$$

By comparison of (17) and (18), we may now state:
A velocity system for the speed $\dot{s}_0$ *in a conservative field with work function* W_1 *is a system of* (1) *trajectories,* (2) *brachistochrones,* (3) *catenaries for the constant of energy h in a conservative field with work function* W_2, *where*

$$(1)\ W_2 = e^{2W_1/\dot{s}_0^2} - h, \qquad (2)\ W_2 = e^{-(2W_1/\dot{s}_0^2)} - h,$$
$$(3)\ W_2 = e^{W_1/\dot{s}_0^2} - h.$$

Since $W_1 =$ constant gives $W_2 =$ constant, the two fields have the same equipotential hypersurfaces and the same lines of force.

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AUGUSTUS DE MORGAN ON DIVERGENT SERIES.

BY PROFESSOR FLORIAN CAJORI.

(Read before the San Francisco Section of the American Mathematical Society April 10, 1920.)

SEVERAL English mathematicians writing in the second quarter of the nineteenth century disapproved of the banishment of divergent series which had been brought about by the followers of A. L. Cauchy and N. H. Abel. These protests were unheeded, doubtless because they were not accompanied by indications disclosing how divergent series could be used with safety. There was one exception, however: Augustus De Morgan reached results which, had they been followed up promptly, might have re-introduced divergent series thirty years earlier than was actually the case. De Morgan's researches have been overlooked in historical statements, except by H. Burkhardt,* who, however, missed the parts of De Morgan which foreshadow a new theory.

* H. Burkhardt "Ueber den Gebrauch divergenter Reihen in der Zeit von 1750–1860," *Math. Annalen*, vol. 70 (1911), pp. 169–206. This article contains much minute information regarding many writers.