NOTE ON RIEMANN SPACES.

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1. IT is proposed to establish the theorem that every closed orientable n-dimensional manifold can be represented on an n-dimensional hypersphere as a Riemann space, or generalized Riemann surface. The argument will be carried through explicitly for the case $n = 3$ only, since the extension to higher dimensions is perfectly automatic and requires scarcely more than the modification of a few subscripts.

2. We know that a 3-dimensional manifold can always be built up out of the points and boundary points of a finite number of tetrahedral regions by suitably matching together in pairs the triangular faces of the bounding tetrahedra. Let $A_1, A_2, \cdots, A_k$ be the points of the manifold which correspond to the vertices of the tetrahedra. Then it can always be so arranged that the four points A_{i1}, A_{i2}, A_{i3}, and A_{i4} corresponding to the vertices of a tetrahedron are distinct, in which case, the bounded tetrahedral region may be designated by the symbol $|A_{i1}A_{i2}A_{i3}A_{i4}|$, where the ordering of the letters A_{ij} is immaterial.

We shall say that every permutation of the letters in the symbol for a tetrahedral region determines a sense on the region and that two permutations determine the same or opposite senses according as they differ by an even or an odd number of transpositions. A tetrahedral region becomes sensed, or oriented, by association with one of the permutations and is then conveniently designated either by the symbol for the sense-giving permutation or by the symbol for any other permutation taken with a plus or minus sign according as the second permutation differs from the first by an even or an odd number of transpositions. Thus, we have

$$A_1A_2A_3A_4 = - A_2A_3A_4A_1 = A_3A_4A_1A_2 = \cdots,$$

but

$$A_1A_2A_3A_4 \neq A_2A_3A_4A_1, \text{ etc.}$$

3. Now, consider two tetrahedral regions $|AA_1A_2A_3|$ and $|A'A_1A_2A_3|$ with a face $A_1A_2A_3$ in common. If senses be