A PROPERTY OF PERMUTATION GROUPS ANALOGOUS TO MULTIPLE TRANSITIVITY.

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THE binary forms

$$a_0z_1^n + a_1z_1^{n-1}z_2 + a_2z_1^{n-2}z_2^2 + \cdots + a_nz_2^n$$

and

$$b_0z_1^n + b_1z_1^{n-1}z_2 + b_2z_1^{n-2}z_2^2 + \cdots + b_nz_2^n$$

are said to be apolar* if

$$(1) \quad a_0b_n - \binom{n}{1}a_1b_{n-1} + \binom{n}{2} a_2b_{n-2} + \cdots + (-1)^n a_nb_0 = 0.$$

Let us assume that $a_0 \neq 0$ and $b_0 \neq 0$; and let the sets of roots of the two corresponding non-homogeneous† equations

$$a_0z^n + a_1z^{n-1} + \cdots + a_n = 0$$

and

$$b_0z^n + b_1z^{n-1} + \cdots + b_n = 0$$

be respectively $x_1, x_2, \cdots, x_n$ and $y_1, y_2, \cdots, y_n$. Then the condition for apolarity may be expressed in terms of these roots as follows:

$$(2) \quad \begin{aligned} S_n - \binom{n}{1}^{-1}S_{n-1}\Sigma_1 + \binom{n}{2}^{-1}S_{n-2}\Sigma_2 + \cdots \\ + (-1)^{n-1}\binom{n}{1}^{-1}S_1\Sigma_{n-1} + (-1)^n\Sigma_n = 0, \end{aligned}$$

where the S's and Σ's are the usual symmetric functions of the y's and x's respectively.

If now we write the product of the n differences

$$(x_1 - y_1)(x_2 - y_2) \cdots (x_n - y_n),$$

permute the x's in all possible ways (keeping the y's fixed), and sum the $n!$ products thus obtained, we get a result which is the same as the left-hand member of (2) except for the

* Cf. Grace and Young, Algebra of Invariants, p. 213; or Dickson, Algebraic Invariants, p. 78.

† This change to non-homogeneous form is only for convenience of expression, and is not essential to the argument.