PARAMETRIC EQUATIONS OF THE PATH OF A PROJECTILE WHEN THE AIR RESISTANCE VARIES AS THE nTH POWER OF THE VELOCITY.

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THE differential equations to be solved are

$$(1)\qquad \frac{W}{g}\frac{d^2x}{dt^2} = -Kv_c^n\frac{dx}{ds}, \qquad \frac{W}{g}\frac{d^2y}{dt^2} = W - Kv_c^n\frac{dy}{ds},$$

in which v_c is the velocity along the path, W is the weight of the projectile and K and n are experimental constants, the dimensions of K being $W \cdot l^{-n} \cdot t^n$. Obviously the X axis is taken horizontal, the Y axis vertically downward.

M. de Sparre* gives a solution for $n = 2$, making however certain approximations in the early stages, and presents his results in two cases corresponding to the paths before and after the time when the slope is unity. Greenhill† treats with much detail the case of $n = 3$.

For the general case of unrestricted n, equations (1) may be written

$$(2)\qquad \begin{aligned} \frac{W}{g}\frac{d^2x}{dt^2} &= -K\left[\left[\frac{dx}{dt}\right]^2 + \left[\frac{dy}{dt}\right]^2\right]^{(n-1)/2} \cdot \frac{dx}{dt}, \\ \frac{W}{g}\frac{d^2y}{dt^2} &= W - K\left[\left[\frac{dx}{dt}\right]^2 + \left[\frac{dy}{dt}\right]^2\right]^{(n-1)/2} \cdot \frac{dy}{dt}, \end{aligned}$$

and next transformed by writing

$$(3)\qquad \frac{dx}{dt} = v = r\cos\theta, \qquad \frac{dy}{dt} = u = r\sin\theta,$$

so that r and θ are the polar coordinates of the hodograph. If the origin is taken at the point of release of the projectile and α is the angle of depression, V being the initial velocity,

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* *Comptes Rendus*, volume 160, p. 584.
† Elliptic Functions, pp. 244–53.