A SET OF COMPLETELY INDEPENDENT POSTULATES FOR THE LINEAR ORDER η*.

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(Read before the American Mathematical Society September 4, 1919.)

PROFESSOR E. V. HUNTINGTON has published† three sets of completely independent postulates for serial order. His set A involves four postulates, which is as high a number of postulates as had been proved completely independent. In the present paper are given seven postulates which form a categorical and completely independent set for the linear order.

Our basis is a class of elements $[p]$ and an undefined dyadic relation (called 'less than') among the elements. If we are given two elements p_1p_2 and if the relation p_1 less than p_2 holds, we will symbolize it by $p_1 < p_2$. If the relation p_1 less than p_2 does not hold, we will symbolize it by $p_1 \nless p_2$.

Our postulates are:

I. If $p_1 < p_2$, then $p_2 \nless p_1$.

II. If $p_1 \nless p_2$, then $p_2 < p_1$; p_1, p_2 distinct.

III. If $p_1 < p_2$ and $p_2 < p_3$, then $p_1 < p_3$.

IV. If $p_1 < p_2$, then there exists a p_3 such that $p_1 < p_3$ and $p_3 < p_2$.

V. For every p_1 there exists a p_2 such that $p_2 < p_1$.

VI. For every p_1 there exists a p_2 such that $p_1 < p_2$.

VII. The class of elements $[p]$ form a denumerable set.

That the set is categorical follows from the fact that the seven postulates stated are the necessary and sufficient conditions for the linear order η. To show complete independence it will be necessary to cite 128 (2^7) examples showing all possible combinations ($\pm\ \pm\ \pm\ \pm\ \pm\ \pm\ \pm$) of our postulates holding and not holding. This is done by giving eight definitions of $<$, and sixteen sets of points such that each definition is applicable to every one of the sets, and every combination

* The linear order η is an ordered set equivalent to that of all the rational numbers.

† "Sets of completely independent postulates for serial order." This BULLETIN, March, 1917. This paper contains a bibliography of complete independence.