position and the number of the maxima and minima of the curve, and shows that all types actually exist. For example: for all $\delta > 0$ sufficiently small, the curve

$$y = (x + \delta i)(x - \delta i)(x + 1 + \delta^3 i)(x + 1 - \delta^3 i) \cdot (x + 1 + \delta^2 + \delta^5 i)(x + 1 + \delta^2 - \delta^5 i)(x + 1 + \delta^2 + \delta^4)$$

has six extremes which, read for decreasing values of x, are arranged so that the first minimum of y is higher than the second maximum, and the second minimum higher than the third maximum.

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FORM OF THE NUMBER OF SUBGROUPS OF PRIME POWER GROUPS.

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§1. *Introduction.*

It is known that the number of the subgroups of order p^α, p being any prime number, which are contained in any group G is always of the form $1 + kp$. When $k = 0$ for every possible pair of values for α and p the group G must be cyclic and vice versa. There are two infinite systems of groups of order p^m containing separately $p + 1$ subgroups of every order which is a proper divisor of the order of the group, viz., the abelian groups of type $(m - 1, 1)$ and the conformal non-abelian groups.

These two infinite systems are composed of all the groups of order p^m involving separately exactly $p + 1$ subgroups of every order which is a proper divisor of p^m. Moreover, if a group of order p^m, $p > 2$, contains exactly $p + 1$ subgroups of each of the two orders p and p^2 it must contain exactly $p + 1$ subgroups of every order which is a proper divisor of the order of the group, and if a group of order 2^m contains exactly three subgroups of each of the orders 2, 4 and 8 it must also contain exactly three subgroups of every other order which is a proper divisor of 2^m.