A NOTE ON "CONTINUOUS MATHEMATICAL INDUCTION."

BY DR. YUEN REN CHAO.

(Read before the San Francisco Section of the American Mathematical Society April 5, 1919.)

1. *Special case.*—Let the function $f(x)$ be defined in some interval of a real variable x.

Hyp. 1. Let there be a point a in the interval such that $f(a) = 0$.

Hyp. 2. Let there be a constant Δ for the interval, such that $f(x) = 0$ implies $f(x + \delta) = 0$, whenever $0 < \delta \leqq \Delta$.

Then for any b in the interval, where $b > a$, $f(b) = 0$.

Proof.—I. If $b - a \leqq \Delta$, then by Hyp. 2 the conclusion follows.

II. If $b - a > \Delta$, then first apply Archimedes' postulate, that is, there will be an integer n and a fraction $\theta (0 \leqq \theta \leqq 1)$ such that

$$b - a = (n + \theta)\Delta, \qquad \text{or} \qquad b = (a + \theta\Delta) + n\Delta.$$

Next, apply ordinary mathematical induction, thus: By Hyp. 1 and 2, since $\theta\Delta < \Delta$,

$$\therefore \quad f(a + \theta\Delta) = 0.$$

Therefore, by 2, again,

$$f[(a + \theta\Delta) + 1 \cdot \Delta] = 0. \tag{1}$$

By 2, if $f[(a + \theta\Delta) + m \cdot \Delta] = 0$, then

$$f[(a + \theta\Delta) + (m + 1)\Delta] = 0. \tag{2}$$

Hence, combining (1) and (2),

$$f(a + \theta\Delta + n\Delta) = 0,$$

that is,

$$f(b) = 0.$$

2. *General case.*—Let $\varphi(x)$ be any propositional function, defined in some interval of a real variable x.