CIRCLE AND SPHERE GEOMETRY.

A Treatise on the Circle and the Sphere. By JULIAN LOWELL COOLIDGE. Oxford, the Clarendon Press, 1916. 8vo. 603 pp.

AN era of systematizing and indexing produces naturally a desire for brevity. The excess of this desire is a disease, and its remedy must be treatises written, without diffuseness indeed, but with the generous temper which will find room and give proper exposition to true works of art. The circle and the sphere have furnished material for hundreds of students and writers, but their work, even the most valuable, has been hitherto too scattered for appraisement by the average student. Cyclopedias, while indispensable, are tantalizing. Circles and spheres have now been rescued from a state of dispersion by Dr. Coolidge's comprehensive lectures. Not a list of results, but a well digested account of theories and methods, sufficiently condensed by judicious choice of position and sequence, is what he has given us for leisurely study and enjoyment.

The reader whom the author has in mind might be, well enough, at the outset a college sophomore, or a teacher who has read somewhat beyond the geometry expected for entrance. Beyond the sixth chapter, however, or roughly the middle of the book, he must be ready for persevering study in specialized fields. The part most read is therefore likely to be the first four chapters, and a short survey of these is what we shall venture here.

After three pages of definitions and conventions the author calls "A truce to these preliminaries!" and plunges into inversion. Circles orthogonal to a fixed circle are self-inverse, and this property generalized defines anallagmatic curves. A family of such circles will have an anallagmatic envelope, for example. A novel term, deferrent, means the locus of centers of a family of orthogonal circles. In 35 closely linked theorems we advance to mutually tangent circles, and find at once a demonstration of Steiner's favorite theorem on poristic systems of circles, whose concise formulation warrants quotation (page 34): "Given two non-intersecting circles which possess the property that a ring of n circles may be