# THE SELF-DUAL PLANE RATIONAL QUINTIC.

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A SELF-DUAL curve is defined to be a curve which has the same number of cusps and double points as it has inflexional tangents and double tangents respectively; and furthermore there are correlations—including polarities—which send the curve into itself.

Haskell, in this BULLETIN, January, 1917, found the maximum number of cusps of an algebraic plane curve, and enumerated the self-dual curves. The well known binomial curves

$$x_1^n = x_0^{n-r} x_2^r$$

have been extensively studied and shown to be self-dual.* The case of the rational plane quartic has been considered in my dissertation at the Johns Hopkins University.†

We here consider briefly the quintic. Since the class of the curve is to equal the order, we have as the fundamental equation,

$$n = n(n-1) - 2d - 3c,$$

where $d$ is the number of double points and $c$ the number of cusps. Hence we have for the quintic,

$$2d + 3c = 15,$$

an equation which has three solutions, as follows:

$$(1)\quad d = 0, \qquad c = 5,$$

$$(2)\quad d = 3, \qquad c = 3,$$

$$(3)\quad d = 6, \qquad c = 1.$$

Case (3) may arise from the degenerate quintic composed of a conic and a cuspidal cubic.

The second case, that of the rational quintic, is the one to be considered here. Furthermore, we consider the curve which

* Loria, Spezielle Ebene Kurven, p. 308; Wieleitner, Algebraische Kurven, p. 136; Snyder, *American Journal*, vol. 30; Winger, *American Journal*, vol. 36.

† "The self-dual plane rational quartic," Dissertation, Johns Hopkins University, May, 1913.