

conditions. In the present paper he extends this method to double series, and discusses the application of the method to convergent double series. It is found that the summation functions of the familiar methods of Cesàro, Hölder, Borel, LeRoy, Riesz, Vallée-Poussin, etc., can be used in building up summation formulas for double series.

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## ON A CERTAIN GENERATION OF RATIONAL CIRCULAR AND ISOTROPIC CURVES.

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### 1. *Introduction.*

A CIRCULAR curve contains the circular points at infinity, or the isotropic points of the plane, as single, or as singular points. A plane isotropic curve is defined as a curve, all of whose infinite points are absorbed by the isotropic points. The equation of such a curve, which is necessarily of even order, in cartesian coordinates may be written in the form

$$(1) \quad (x^2 + y^2)^k + \varphi(x, y) = 0,$$

in which  $\varphi(x, y)$  is a polynomial of degree  $2k - 1$  at most.

If  $P(\xi, \eta)$  is a fixed point and  $A(x, y)$  any other point so that  $PA = \rho$ , and  $\theta$  the angle between  $PA$  and the positive direction of the  $x$ -axis, then the coordinates of  $A$  are  $x = \xi + \rho \cos \theta$ ,  $y = \eta + \rho \sin \theta$ , and satisfy equation (1) when  $A$  is on the curve. The condition for this is an equation of the form

$$(2) \quad \rho^{2k} + \alpha_1 \rho^{2k-1} + \alpha_2 \rho^{2k-2} + \dots + \alpha_{2k-1} \rho + \alpha_{2k} = 0,$$

in which  $\alpha_1, \alpha_2, \dots, \alpha_{2k-1}$  are coefficients, which, in general, depend on  $\xi, \eta, \theta$  and the coefficients of (1); while  $\alpha_{2k}$  is independent of  $\theta$ . The roots  $\rho_1, \rho_2, \dots, \rho_{2k}$  of (2) are the distances  $PA_i$  ( $i = 1, 2, 3, \dots, 2k$ ) of the points of inter-