ROTATING CYLINDERS AND RECTILINEAR VORTICES.

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§ 1. *Rectilinear Vortex and Rotating Cylinder in a Stream of Incompressible Fluid.*

WE shall assume that the rotation of the cylinder produces a circulation around the cylinder which may be approximately represented by placing a rectilinear vortex along the axis of the cylinder, a device which was adopted by Lord Rayleigh* in his paper "On the irregular flight of a tennis ball." Let V be the velocity of the stream, $2\pi k$ the circulation around the cylinder and $2\pi c$ the strength of the rectilinear vortex at the side of the cylinder. Assuming that the axis of this vortex is parallel to the axis of the cylinder, the motion is two-dimensional and we may represent the velocity potential ϕ and stream line function ψ as follows:

$$\phi + i\psi = U\left(z + \frac{a^2}{z}\right) + ik \log z + ic \log \frac{z - z_0}{z - z_1},$$

where a is the radius of the cylinder, $z = x + iy$ is a complex variable specifying the position of a point relative to the axis of the cylinder, z_0 specifies the position of the vortex, and z_1 that of its image.

Let (u, v) be the component velocities of the fluid at (x, y), (u_0, v_0) the component velocities of the vortex; then differentiating the above equation with regard to z we find that

$$u - iv = U\left(1 - \frac{a^2}{z^2}\right) + \frac{ik}{z} + ic\left[\frac{1}{z - z_0} - \frac{1}{z - z_1}\right].$$

In calculating (u_0, v_0) we ignore the infinite velocity produced by the vortex itself, consequently

$$u_0 - iv_0 = U\left(1 - \frac{a^2}{z_0^2}\right) + \frac{ik}{z_0} - ic\frac{1}{z_0 - z_1}.$$

* *Mess. of Math.* (1878); Scientific Papers, vol. 1, p. 344. See also Lamb's Hydrodynamics, 4th ed., 1916, p. 77; Greenhill, *Mess. of Math.*, vol. 9 (1880), p. 113. Report on Gyroscopic Theory, Report of the Advisory Committee for Aeronautics, No. 146, p. 238.