

of 4, 6, 9, 10, 11, 12, 13, 14, 18, 20, 22, 24, 30, or 72 sides, respectively, with appendices ranging in number from 0 up to more than 100. The distinct covariants for the two noncongruent systems on 13 elements were only nine in number and much simpler in form. We see that an increase in the number  $n$  of elements, which is probably always accompanied by an increase in the number of distinct systems, produces greater complexity in the form and a very rapid increase in the number of distinct covariants connected with the noncongruent systems.

To extend this method of trains to systems on 19 or more elements would be evidently too laborious, if the object is only to classify the different triad systems. Here the analogy of invariants of algebraic forms under linear transformation is instructive; the complete calculation of systems of invariants is always possible, but only desirable when it involves finite time, as in forms of very low order. Beyond that, it is only particular forms with special invariant characters that are of general interest. So here, it is obviously most interesting to give detailed study first to triad systems which have covariant trains ending in polygon-cycles containing the largest possible number of extraneous triads. This recalls Professor E. H. Moore's study of systems whose groups are cyclic and those might probably be found again early in the proposed research.

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## A THEOREM ON AREAS.

BY PROFESSOR TSURUICHI HAYASHI.

THE relative area of two given convex ovals in the same plane, swept out by moving the join of two points lying on the peripheries of the two ovals respectively, so that the point of the join dividing it in a given ratio traces out the periphery of the area containing the totality of all the points which divide the joins of two points lying on and within the two ovals respectively, satisfies the relation

$$\sqrt{S} \leq \sqrt{A} \sim \sqrt{B},$$

independent of the ratio,  $A, B, S$  being the areas of the two ovals and their relative area, respectively.