APPLICATIONS OF THE THEORY OF SUMMABILITY TO DEVELOPMENTS IN ORTHOGONAL FUNCTIONS.

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THE very considerable body of literature which may be described by the above title belongs almost entirely to the present century. Its extent is only roughly indicated by the bibliography at the end of the paper, which makes no pretensions to being complete. The type of series considered here constitutes one of the three most important classes of series to which the theory of summability has been applied, the other two being power series and Dirichlet's series. A noteworthy feature of the applications with which we shall be concerned is found in their usefulness in an important branch of applied mathematics, namely the theory of the flow of heat and electricity.

§1. *The Summability of Fourier's Series.*

The first writer to deal with the topic of this section was Fejér. In his fundamental paper of 1903 [5]* he established among other results the summability $(C1)$ of the Fourier development of an arbitrary function satisfying very wide conditions, at all points where the function is continuous or has a finite jump. We shall give a proof of this theorem, under somewhat modified conditions, which is substantially the same as Fejér's proof.

The Fourier development of $f(x)$ may be written in the form

$$(1)\qquad \frac{1}{2\pi}\int_a^{2\pi+a} f(\theta)d\theta + \frac{1}{\pi}\sum_{n=1}^{\infty}\int_a^{2\pi+a} f(\theta)\cos n(\theta-x)d\theta,$$

it being assumed that $f(x)$ is periodic of period 2π. We have for the sum of the first n terms of (1)

$$(2)\qquad s_n(x) = \frac{1}{\pi}\int_a^{2\pi+a} f(\theta)\frac{\cos(n-1)(\theta-x)-\cos n(\theta-x)}{2(1-\cos(\theta-x))}d\theta.$$

* The numbers in brackets refer to the bibliographical list at the end of the paper.