ω and therefore N being prime to p. We have further

$$(5) \qquad \left[\frac{d^{p-1}(VX)}{dv}\right]_{v=0} \equiv X(1)\left[\frac{d^{p-1}V}{dv}\right]_{v=0} \equiv -X(1) \pmod{p}.$$

Comparison of (5), (4) and (3) gives the theorem (1).

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TRAJECTORIES AND FLAT POINTS ON RULED SURFACES.

BY MR. J. K. WHITTEMORE.

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§1. *Introduction.* In the following paper we determine the flat points* of a ruled surface with real rulings, and prove a new property of the orthogonal trajectories of the rulings. This property may be extended to any isogonal trajectory of the rulings, not itself a ruling, and may be regarded as a generalization of Bonnet's familiar theorem.†

Let S be a ruled surface with real rulings, g any such ruling, C an orthogonal trajectory of the rulings, generally not a straight line; let the coordinates of any point of C be x_0, y_0, z_0, and consider these as functions of v, the arc of C. When C is not a straight line let the direction cosines of its tangent be α, β, γ, of its principal normal be l, m, n, and of its binormal be λ, μ, ν; let R and T be respectively the radii of curvature and torsion of C, ψ the angle measured from the principal normal towards the binormal to the direction chosen as positive on g. We suppose C to be a rectifiable curve generally without singular points in the portion considered; the curvature $1/R$ and the torsion $1/T$ shall have finite first derivatives with respect to v, and ψ shall have a finite second derivative.

The surface S is given by

$$(1) \qquad x = x_0 + uL, \qquad L = l \cos\psi + \lambda \sin\psi,$$

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* Flat points are defined in § 4 of this paper.
† See Eisenhart, Differential Geometry, p. 248.