Hence

$$h_{n+1} \leqq h_n,$$

and h_n decreases monotonically.

Physical Interpretation.—A physical interpretation of the problem just discussed is given by the small oscillations of a simple pendulum, the cord of which, in Case I, is gradually lengthened, monotonically, remaining finite; while in Case II the cord is monotonically shortened, never becoming less than a positive fixed length.

The same physical picture is useful, too, for showing that the theorem, stated without restriction on $\varphi(t)$, is false. For the way a child swings higher and higher is to lengthen the equivalent simple pendulum on the downward arc, and suddenly to shorten it as the upward arc begins.

Bôcher's theorem (*b*), l. c., namely that, when $\varphi(t)$ is continuous and

$$0 < g \leqq \varphi(t) \leqq G,$$

there is one and only one solution which remains finite, and this vanishes at infinity, is correct without any further restriction on the function φ. An elementary analytic proof can be given by the aid of the law of the mean and Taylor's theorem with the remainder, carried through the term of the second order.

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PROOF OF A PROPERTY OF THE NORM OF A CYCLOTOMIC INTEGER.

BY MR. H. S. VANDIVER.

(Read before the American Mathematical Society April 27, 1918.)

KUMMER in his first proof of the general law of reciprocity between two ideals in a regular cyclotomic algebraic field gave a theorem* which forms a link in his chain of reasoning. Let

* *Abhandlungen Berlin Academy*, 1859, p. 119, formula (7).