CONTINUOUS SETS THAT HAVE NO CONTINUOUS SETS OF CONDENSATION.

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JANISZEWSKI has shown* that if A and B are two distinct points then every bounded set of points that is *irreducibly* continuous† from A to B, and has no continuous set of condensation,† is a simple continuous arc from A to B. In the present paper I will establish the following result.

THEOREM. *Every bounded continuous set of points that has no continuous set of condensation is a continuous curve.*‡

Proof. Suppose M is a bounded continuous set of points that has no continuous set of condensation. It has been shown by Hahn§ that every bounded continuous set of points that is connected "im kleinen" is a continuous curve. I shall proceed to show that the point set M is connected "im kleinen." Suppose that it is not. Then there is a point P belonging to M and a circle K with center at P such that within every circle whose center is P there exists a point which does not lie together with P in any connected subset of M that lies

* S. Janiszewski, "Sur les continus irréductibles entre deux points," *Journal de l'Ecole Polytechnique*, 2e série, vol. 16 (1911–12), pp. 79–170.

† A set of points is said to be *connected* if, however it be divided into two mutually exclusive subsets, one of these subsets contains a limit point of the other one. A set of points is said to be *continuous* if it is closed and connected and contains more than one point. A continuous set of points containing the two distinct points A and B is said to be *irreducibly continuous* from A to B if it contains no other continuous set that contains both A and B. The continuous set N is said to be a *continuous set of condensation* of the continuous set M if N is a subset of M and every point of N is a limit point of $M-N$.

‡ A *continuous curve* is the set of all points $\{(x, y)\}$ satisfying the equations $x = f_1(t)$, $y = f_2(t)$ $(0 \leqq t \leqq 1)$, where $f_1(t)$ and $f_2(t)$ are continuous functions of t. In case there do not exist, in the interval $(0 \leqq t \leqq 1)$, two distinct numbers t_1 and t_2 such that $f_1(t_1) = f_1(t_2)$ and $f_2(t_1) = f_2(t_2)$, then this curve is a *simple* continuous arc.

§ Hans Hahn, "Ueber die allgemeinste ebene Punktmenge, die stetiges Bild einer Strecke ist," *Jahresbericht der Deutschen Mathematiker-Vereinigung*, vol. 23 (1914), pp. 318–322. A set of points M is said to be *connected "im kleinen"* (cf. Hahn, loc. cit.) if for each point P of M and each circle K with center at P there exists, within K, another circle K', with center at P, such that if X is a point of M within K' then X and P lie together in some connected subset of M that lies entirely within K.