For an even λ, this becomes

$$\alpha - \alpha_2 + \alpha_4 - \cdots = 1.$$

For any λ, there is $\alpha_1 - \alpha_3 + \alpha_5 - \cdots = 0$. When λ is odd, then $-\alpha + \alpha_2 - \alpha_4 + \cdots = 1$. When n is odd, say $n = 2\lambda + 1$, then $\alpha - \alpha_2 + \alpha_4 - \cdots = 0$, and $\alpha_1 - \alpha_3 + \alpha_5 - \cdots = 1$, when λ is odd; $-\alpha_1 + \alpha_3 - \alpha_5 + \cdots = 1$, when λ is even.

We shall, in particular, consider the case where (15) has the form

$$w^n - v^{2k} \cdot u^{n-2k} = 0, \tag{16}$$

in which n and k must both be either even or odd in order that (16) may reduce to (14) for $u = -1, v = -i$.

After a rather complicated process of elimination the cartesian equation of this special class of curves with the nth roots of unity as foci becomes

$$x^{n-2k} \cdot y^{2k} = (-1)^{n-2k} \cdot \frac{(2k)^{2k}}{n^n \cdot (n-2k)^{2k-n}}, \tag{17}$$

which is an n-ic. For $n = 3, k = 1$, we get the cubic hyperbola $xy^2 = -4/27$. The condition for a proper n-ic in (17) is evidently $n - 2k \geqq 1, n \geqq 3$.

UNIVERSITY OF ILLINOIS.

QUADRATIC SYSTEMS OF CIRCLES IN NON-EUCLIDEAN GEOMETRY.

BY PROFESSOR D. M. Y. SOMMERVILLE.

(Read before the American Mathematical Society, October 26, 1918.)

§ 1. The equation of any circle can be written

$$kS - \alpha^2 = 0, \tag{1}$$

where $S = 0$ is the equation of the absolute, and

$$\alpha \equiv lx + my + nz = 0$$

is the equation of the axis, the center being the absolute polar of this line.