# GENERAL ASPECTS OF THE THEORY OF SUMMABLE SERIES.

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## §1. *General Considerations Relating to the Sum of an Infinite Series.*

IN 1811 Fourier* read before the Paris Academy a memoir which contained an acceptable definition of convergence of an infinite series; but this work remained unpublished for eight or nine years. In 1817 Bolzano stated a precise definition of convergence. Independently in 1821 Cauchy also formulated the definition in an exact manner. He and Abel insisted so forcefully upon the necessity of the distinction between convergence and divergence and the danger in employing divergent series that the latter came into such disrepute as not to be studied systematically for nearly three quarters of a century. For a long time no one saw how to obviate the difficulties pointed out so incisively by those who first recognized the pitfalls in the use of divergent series. And yet both Abel and Cauchy, the leading instigators, had misgivings† as to the justice of the decision by which these series were banished from the mathematical community and they were given up as friends who had done some things well but could not be trusted because they had also done some things ill.‡

Certain difficulties, however, still remain when one tries to treat convergent series independently of any reference to divergent series, as we shall show more fully in a moment.

In the first attempt to formulate a suitable definition of

* For references relating to the first paragraph see Encyclopédie des Sciences mathématiques, I, $1_2$, pp. 211–214.

† See quotations in Bromwich's Infinite Series, 1908, p. 264. Indeed Cauchy himself showed how the celebrated series of Stirling in the theory of the gamma function could be used in a legitimate way for purposes of numerical computation.

‡ An interesting and valuable discussion of several topics in the theory of divergent series and continued fractions will be found in Van Vleck's lectures at the Boston Colloquium in 1903, published in 1905. In these lectures some topics are treated to which we do not refer in this paper.