THE INTRINSIC EQUATION FOR EULER'S RESISTANCE INTEGRAL.

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EULER, in his Scientia Navalis, Proposition 53, gives a problem whose solution leads to the resistance integral

$$I = \int_{t_0}^{t_1} \frac{y'^3}{x'^2 + y'^2} dt. \tag{1}$$

This problem is as follows: Among all curves AM which with the axis AP and perpendicular PM comprehend the same area; to find that one which with its symmetric branch on the opposite side of the axis AP will form the figure offering the least resistance in water, when it moves in the direction PA along the axis.

This is a problem of the isoperimetric type where the integral I to be minimized is the integral (1) and the condition imposed is that the area between the axis and the curve is fixed, i. e.,

$$G = \int_{t_0}^{t_1} yx'dt = l.$$

The solution of this problem is known to be

$$x = \frac{1}{\lambda} \frac{p^2 - 1}{(p^2 + 1)^2} + \alpha, \quad y = \frac{1}{\lambda} \frac{2p^3}{(p^2 + 1)^2} + \beta,$$

where $p = y'/x'$ and α, λ and β are constants. Since this curve can always be obtained by a similarity transformation from the curve

$$x = \frac{p^2 - 1}{(p^2 + 1)^2}, \quad y = \frac{2p^3}{(p^2 + 1)^2}, \tag{2}$$

the general properties of the extremal can be obtained from the latter.

So far as is known, the exact nature of the extremals has never been determined. It is simply stated that they are rational curves of the fourth order with three cusps. The object of this paper is to show that when their intrinsic