NOTE CONCERNING THE NUMBER OF POSSIBLE INTERPRETATIONS OF ANY SYSTEM OF POSTULATES.

BY PROFESSOR C. J. KEYSER.

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IN this note it is assumed that a mathematical system of postulates contains one or more undefined terms and that at least one of these denotes an element, i. e., a thing as distinguished from a relation. The assignment of an admissible meaning, or value, to each of the undefined elements of a postulate system will be spoken of as an interpretation of the system. By "admissible" meanings are meant meanings that satisfy the postulates or that, in other words, render them true propositions.

A postulate system may be such that from a given interpretation of it a second interpretation may be derived, from the second a third one, from the third a fourth, and so on, in such a way that mathematical induction is available for proving that the system admits of a denumerable infinitude of different interpretations. Such a system, for example, is Hilbert's system for euclidean metric geometry, where the undefined element-names are point, line, and plane. It is well known that one interpretation of this system results from allowing the term point to mean an ordered triad (x, y, z) of real numbers, the term plane to mean the class of triads satisfying an equation of the form $Ax + By + Cz + D = 0$ where not all the coefficients vanish, and the term line to mean the class of triads satisfying a pair of such equations.

If now we let the equations

$$(1) \quad \pi_1 + \lambda\pi_2 = 0, \qquad \pi_3 + \mu\pi_4 = 0, \qquad \pi_5 + \nu\pi_6 = 0$$

represent three pencils of planes, where the term plane has the meaning above assigned, it is evident that we may get a second interpretation of the Hilbert system by agreeing that the term point shall mean a triad (λ, μ, ν) of planes selected as indicated from the given pencils, that by a plane shall be meant a congruence, $a\lambda + b\mu + c\nu + d = 0$, of such triads,