SURFACES OF REVOLUTION IN THE THEORY OF LAMÉ'S PRODUCTS.

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IN *Berliner Monatsberichte*, February, 1878, Wangerin discussed the problem of the most general orthogonal surfaces of revolution such that if Laplace's equation be written in coordinates corresponding to these surfaces a solution may be obtained in the form of a Lamé's product with an extraneous factor, i. e.,

$$V = \lambda \cdot R_1 \cdot R_2 \cdot \theta. \tag{1}$$

R_1, R_2, and θ are functions respectively of the parameters of the two families of surfaces and the meridian planes, while λ may involve all three parameters. Wangerin showed that λ is $1/\sqrt{r}$, where r is the distance from the axis of revolution to the point of intersection of the three surfaces. He also deduced certain meridian curves which have been discussed by Haentzschel in Reduction der Potentialgleichung, Berlin, 1893.

In reducing Laplace's equation the terms involving θ are removed in the usual manner, leaving a partial differential equation which, as both writers state, is resolvable into two ordinary differential equations provided F and F_1, its conjugate, can be found which shall satisfy

$$\frac{F'(t+iu)\cdot F_1'(t-iu)}{[F(t+iu)-F_1(t-iu)]^2} = H(t) + K(u). \tag{2}$$

After F is found the two families of meridian curves follow from

$$x + ir = F(t+iu),\ x - ir = F_1(t-iu),\ r = \sqrt{(y^2+z^2)}, \tag{3}$$

by elimination of u and t respectively, thus making t and u the parameters of which R_1 and R_2 are functions as stated above. As will be seen, H and K disappear in the process of finding F, and the principal result in this paper is a direct process of computing H and K, avoiding the method of