

$y_1, xy_1, \dots, x^{r-1}y_1$  are all solutions while  $x^r y_1$  is not a solution. If  $r$  is greater than unity, the solution is said to be repeated. If  $y_1$  is a repeated solution, then it must also satisfy the equation

$$na_0 D^{n-1}y + (n-1)a_1 D^{n-2}y + \dots + a_{n-1}y = 0,$$

that is, the equation obtained from (1) by formal differentiation with respect to  $D$ . The first elements of the theory of repeated solutions of (1) and a certain more general class of equations thus suggested is developed on a simple postulational basis.

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*Secretary of the Section.*

## ELEMENTARY INEQUALITIES FOR THE ROOTS OF AN ALGEBRAIC EQUATION.

BY PROFESSOR R. D. CARMICHAEL.

(Read before the American Mathematical Society, October 27, 1917.)

1. LET us write the general algebraic equation in each of the following forms:\*

$$\begin{aligned} x^n &= a_1 x^{n-1} + a_2 x^{n-2} + a_3 x^{n-3} + \dots + a_n^n, \\ (1) \quad x^n &= c_{n1} \alpha_1 x^{n-1} + c_{n2} \alpha_2 x^{n-2} + \dots + c_{nn} \alpha_n^n, \\ x^n &= \beta_1 x^{n-1} + \beta_2 x^{n-2} + \dots + \beta_n, \end{aligned}$$

where

$$a_i^i = c_{ni} \alpha_i^i = \beta_i \quad (i = 1, 2, \dots, n),$$

and  $c_{n1}, c_{n2}, \dots, c_{nn}$  denote the binomial coefficients for the power  $n$ .

If we let  $X$  denote the greatest absolute value of a root of equation (1) and let  $\alpha$  denote the greatest absolute value of the quantities  $|\alpha_1|, |\alpha_2|, \dots, |\alpha_n|$ , then, as was shown by Carmichael and Mason,† we have  $X \geq \alpha$ , the equality sign

\* The fruitful and convenient notation employed in the first equation was suggested to me by my friend and colleague, Dr. A. J. Kempner.

† This BULLETIN, vol. 21 (1914), pp. 14-22. Carmichael and Mason stated the theorem for the equation whose roots are the reciprocals of those of (1).