corresponds to the other ray tangent, and recalling Wilczynski's theorem, we are led at once to the final form in which we stated the geometric characterization of conjugate nets with equal point invariants. This characterization may of course be established analytically, and independently of Wilczynski's theorem, so that, on the basis of our general theorem, Wilczynski's characterization may then be obtained from ours.

HARVARD UNIVERSITY,
August 22, 1917.

ON THE DIFFERENTIABILITY OF ASYMPTOTIC SERIES.

BY DR. J. F. RITT.

(Read before the American Mathematical Society, October 27, 1917.)

THE question of the differentiability of asymptotic series seems not to have received adequate treatment.* Writers on the theory of asymptotic convergence content themselves always with stating that if $P(x)$ has the asymptotic representation

$$P(x) \sim a_0 + a_1x + a_2x^2 + \cdots + a_nx^n + \cdots,$$

it may not have a derivative at all, and that even if a derivative does exist, the derivative may not admit of asymptotic development.

A failure to distinguish between the real and complex domains, in this connection, is responsible for a serious lacuna, which it is the purpose of this note to fill.

Let $P(x)$, defined in a sector with vertex at the origin, be analytic within the sector, in the neighborhood of the origin, and continuous on the sides of the sector, and at the origin. Let $P(x)$ have the asymptotic development of finite order

$$P(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n + \epsilon(x)x^n,$$

where $n \geqq 2$ and where $\epsilon(x)$ goes to zero with x. We say that

* Since writing this note, I have ascertained by correspondence with Professor Birkhoff that he was familiar with and had used the result given here, but failed to publish it, being under the impression that it was contained in an article by W. B. Ford (*Bull. de la Soc. Math. de France*, 1911, p. 347).