FINITE GROUPS.

Theory and Applications of Finite Groups. By G. A. MILLER, H. F. BLICHFELDT, and L. E. DICKSON. New York, Wiley, 1916. 8vo. 17 + 390 pp.

DURING the last half-century the theory of groups has been explored with great vigor and success, and it is now an imposing body of doctrine. Its importance is principally due to the fact that it enters into the very warp and woof of a large number of mathematical textures. "Many branches of mathematics are nothing but the theory of invariants of special groups." This is especially true of geometry, as was made clear once for all by Klein. Geometry is not one science, but an infinite variety of sciences, depending on the choice of the group on which it is based. As the fundamental group becomes larger and more extensive, the corresponding geometry becomes less extensive and more fundamental. Thus, beginning with the elementary geometry of congruent figures, we obtain in succession a number of different geometries among which are the geometry of similar figures, affine geometry, projective geometry, conformal geometry, and analysis situs.

From a purely abstract standpoint groups may be classified in various ways: they are either abelian or non-abelian, finite or infinite, continuous or discontinuous. The principal classes of concrete groups are permutation groups, linear groups, and groups of non-linear transformations, together with numerous geometrical and mechanical applications. Even in the study of point sets and other aggregates groups have been found useful; for instance, in Bernstein's proof that if two cardinal numbers $n\alpha$ and $n\beta$ are equal, then $\alpha = \beta$.

In a linear group the transformations may be homogeneous or fractional, singular or non-singular, but the most striking principle of classification is that which depends on the nature of the coefficients, which may belong to an infinite field, a finite field, or no field whatever. If the coefficients belong to an infinite field or domain, for instance that of all complex numbers, we obtain a most important class of groups, both continuous and discontinuous, with which are closely asso-