A CONTINUOUS FUNCTION WHOSE DEVELOPMENT IN BESSEL'S FUNCTIONS IS NON-SUMMABLE OF CERTAIN ORDERS.

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THE first example of a continuous function whose Fourier's development diverges at one or more points was given, as is well known, by Du Bois-Reymond.* Much more recently Haar† showed how to construct a continuous function whose Sturm-Liouville development is divergent, and a continuous function whose development in Legendre's functions has this same property. Somewhat later Gronwall‡ gave in explicit form a function whose development in Legendre's functions is not summable (Ck) for $0 \leqq k \leqq \frac{1}{2}$, at the point $x = 1$.

As far as the writer is aware, no examples have thus far been given of continuous functions whose developments in Bessel's functions diverge or fail to be summable of certain orders at one or more points. In the present paper there is exhibited in explicit form a continuous function whose development in Bessel's functions of order zero is not summable (Ck) for $0 \leqq k < \frac{1}{2}$, at the point $x = 0$. This function is analogous to a function given by Fejér§ whose Fourier development diverges for $x = 0$. The proof of the non-summability, however, follows different lines from Fejér's proof of the divergence for his example.

If we represent by A_r the rth coefficient in the development of an arbitrary function $f(x)$ in terms of Bessel's functions, we have

$$(1) \qquad A_r = \frac{\int_0^1 xf(x)J_0(\lambda_r x)dx}{\int_0^1 xJ_0^2(\lambda_r x)dx},$$

where λ_r is the rth root of the equation

* *Abhandlungen der Bayerischen Akademie der Wissenschaften*, vol. 12 (1876).

† Inaugural Dissertation, Göttingen (1909); also *Math. Annalen*, vol. 69 (1910).

‡ *Math. Annalen*, vol. 75 (1914).

§ *Journal für die reine und angewandte Mathematik*, vol. 137 (1910).