NOTE ON THE PARAMETRIC REPRESENTATION OF AN ARBITRARY CONTINUOUS CURVE.

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A CONTINUOUS curve may be defined by a pair of equations

$$x = f(t), \qquad y = \varphi(t),$$

where the functions f and φ are defined and continuous throughout some interval, say for $0 \leqq t \leqq 1$, and are not both constant throughout that interval. It is sometimes an aid to the imagination to suppose that they are not simultaneously constant throughout any subinterval of their interval of definition. The question naturally arises, whether this implies a restriction on the curve itself, or merely on the particular parametric representation employed. It is obvious that if there are only a finite number of intervals where f and φ are constant together, a suitable change of parameter will eliminate such intervals entirely. The present note offers a proof that this is true in all cases. A decidedly different proof has been given by Fréchet;* it is believed that the method of treatment here adopted is considerably more elementary than his, as far as obtaining the indicated result is concerned, though it does not yield certain additional knowledge which is incidental to his presentation.

On careful scrutiny, it appears that the problem of a satisfactory definition of the identity of two curves is not an altogether simple one;† to avoid a discussion of this point, the result is stated in the following form, which is free of ambiguity:

THEOREM. *Let a pair of functions*

$$x = f(t), \qquad y = \varphi(t) \tag{1}$$

be defined and continuous for $0 \leqq t \leqq 1$. *Then it is possible to*

* Fréchet, "Sur quelques points du calcul fonctionnel," *Rendiconti del Circolo Matematico di Palermo*, vol. 22 (1906), pp. 1–74; pp. 58–59, 67–70.

† Cf. Fréchet, loc. cit., pp. 51–53.