IRRATIONAL TRANSFORMATIONS OF THE GENERAL ELLIPTIC ELEMENT.

BY PROFESSOR F. H. SAFFORD.

(Read before the American Mathematical Society, April 28, 1917.)

In 1865 G. G. A. Biermann published as derived from Weierstrass's lectures the following formula:

$$(1)\quad F(x) = x_0 + \frac{\sqrt{R(x_0)}\,\sqrt{S} + \frac{1}{2}R'(x_0)[s - \frac{1}{24}R''(x_0)] + \frac{1}{24}R(x_0)R'''(x_0)}{2\left[s - \frac{1}{24}R''(x_0)\right]^2 - \frac{1}{2}A\cdot R(x_0)}.$$

F is the solution of

$$(2)\quad (F')^2 = AF^4 + 4BF^3 + 6CF^2 + 4B'F + A' = R(F).$$

The accents used with F and R denote differentiation, x_0 is an arbitrary constant, and A, B, C, B', A' are constant coefficients. Also

$$(3)\quad S = 4s^3 - g_2 s - g_3 = 4(s - \epsilon_1)(s - \epsilon_2)(s - \epsilon_3),$$

$$s = \wp(x), \quad g_2 = AA' + 3C^2 - 4BB',$$

$$g_3 = ACA' + 2BCB' - AB'^2 - A'B^2 - C^3.$$

In Enneper, Elliptische Functionen, Greenhill, Elliptic Functions; and Haentzschel, Reduction der Potentialgleichung, are several applications of this formula.

Replacing x by $t + iu$ and F by $x + iy$ in the preceding equations gives

$$(4)\qquad x + iy = \varphi(t + iu),$$

from which may be obtained in the usual manner two mutually orthogonal families of curves having t and u as parameters. For several cases this has been done by Haentzschel, and the writer has given discussions and extensions in *Archiv der Mathematik und Physik.* It is evident that (1) becomes very simple when x_0 is a root of $R(x) = 0$, for which Haentzschel's