Also, the determinant of equations (7)–(9) vanishes, as may be seen at once from the fact that (9) may be obtained by subtracting β' times (7) from β times (8). Hence* we have

THEOREM II. *The parameters of the points of contact of the three pairs of tangents that can be drawn to the R^3 from three collinear points of the R^3 are harmonic to the same quadratic, or form a set in involution.*

Another result which may be derived as a corollary of Theorem I we shall state as

THEOREM III. *Lines on a point P of an R^3 cut the R^3 in pairs of residual points whose parameters are harmonic to the parameters of the points of contact of the two additional tangents drawn to R^3 from P.*

Although Theorem III may be regarded a corollary of Theorem I, it may be established independently. Thus: Let $P(d_0, d_1, d_2)$ be the point and $(\kappa x) \equiv \kappa_0 x_0 + \kappa_1 x_1 + \kappa_2 x_2 = 0$ any line on P. Then $(\kappa d) = 0$. The parameters of the residual points cut out of (1) by $(\kappa x) = 0$ are the roots of

$$(10) \qquad (\kappa a)t^2 + 3(\kappa b)t + 3(\kappa c) = 0$$

and (10) is apolar to (8), for

$$3(\kappa c)\beta + 3(\kappa a)\alpha' - 3(\kappa b)\beta' = 0,$$

as may be shown from (3) and the fact that $(\kappa d) = 0$.

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EXAMPLES OF A REMARKABLE CLASS OF SERIES.

BY PROFESSOR R. D. CARMICHAEL.

(Read before the American Mathematical Society, April 28, 1917.)

Two-Fold and One-Fold Expression of the Properties of Functions.

1. IN the development of analysis during the past generation it has frequently happened that functions have arisen which are analytic in a sector of the complex plane and in

* Salmon, Higher Algebra, fourth edition, p. 180.