THE PROJECTION OF A LINE SECTION UPON THE RATIONAL PLANE CUBIC CURVE.

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(Read before the American Mathematical Society, April 28, 1917.)

Introduction.

THE rational plane curve of the third order, which we shall refer to as the R^3, is of the fourth class; that is, from an arbitrary point of the plane four tangents can be drawn to the curve. But if the point is selected on the R^3 itself, the tangent at the point accounts for two of these tangents, and, therefore, from such a point only two additional tangents can be drawn to the curve. A line section yields three points of the R^3 and these, in the manner just described, determine three pairs of additional tangents. An investigation of the points of a line and the six tangents so determined shows that the relations which exist among these are interesting as well as of a fundamental character.

We shall let

$$x_i = a_it^3 + 3b_it^2 + 3c_it + d_i \qquad (i = 0, 1, 2) \tag{1}$$

be the parametric equations of the points of the R^3, and it has been found convenient to use the following abbreviations:

$$\alpha = | abc |, \quad \beta = | abd |, \quad \beta' = | acd |, \quad \alpha' = | bcd |. \tag{2}$$

Also, it may be verified that the identities

$$a_i\alpha' - b_i\beta' + c_i\beta - d_i\alpha = 0 \tag{3}$$

exist among the coefficients in (1) and the Greek letters of (2).

The Choice of a Line Section.

As the parameters 0 and ∞ may be assigned to any two elements in a one-dimensional space, we select the line determined by the points of the R^3 whose parameters are 0 and ∞. From (1) it follows that the coordinates of these points are d_i and a_i, respectively; hence the equation of the line determined by them is $| adx | = 0$, and the parameter of the third point