SINGULAR POINTS OF ANALYTIC TRANSFORMATIONS.

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LET a transformation be defined by the equations

$$x_i = \varphi_i(u_1, \ldots, u_n) \qquad (i=1, \ldots, n),$$

where the functions φ_i are meromorphic in the origin and the first μ of them, $0 < \mu \leqq n$, have a non-essential singularity of the second kind there, the remaining $n - \mu$ functions being analytic or having a pole there. Let the point (x) be interpreted in the space of analysis.

Let those points of the region

$$|u_k| < \eta_k \qquad (k = 1, \ldots, n),$$

where η_k is a positive number, in which no function φ_i has a non-essential singularity of the second kind, be denoted by $\mathfrak{T}$, and let the points (x) which form the images of the points (u) of $\mathfrak{T}$ constitute the region M. There will be certain points of the (x)-space which will lie on the boundary of M, no matter how far $\mathfrak{T}$ is restricted. The manifold of these points shall be denoted by $\mathfrak{M}$.

The object of this note is to communicate the following theorem, the proof of which will shortly be published elsewhere.

THEOREM. *The manifold $\mathfrak{M}$ is made up of a finite number of algebraic manifolds of the following kind:*

In the space of the first μ variables $(x_1, \ldots, x_\mu)$ there exists a manifold $\mathfrak{K}$ formed by a finite number of irreducible algebraic curves ($k = 1$), surfaces ($k = 2$), or hypersurfaces of order $k < \mu$, the number k being the same for all; or finally, if $\mu < n$, $\mathfrak{K}$ may include all the points of the space in question, and we set here $k = \mu$.

Then $\mathfrak{M}$ consists of the points $(x_1, \ldots, x_n)$, where $(x_1, \ldots, x_\mu)$ is an arbitrary point of $\mathfrak{K}$, and

$$x_j = a_j = \frac{g_j(0, \ldots, 0)}{G_j(0, \ldots, 0)} \qquad (j = \mu + 1, \ldots, n).$$

HARVARD UNIVERSITY,
April, 1917.