EQUILONG INVARIANTS AND CONVERGENCE PROOFS.

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THE writer has studied the invariants of a pair of analytic curves under the equilong group with the main object of throwing light on the corresponding question in the more important conformal geometry.* The two theories present many analogies, but are not connected by a strict principle of duality. The number of invariants and their orders turn out to be the the same, though the results have to be calculated independently.

In some questions, however, the two theories differ essentially, not only in the methods to be employed, but also in the results obtained. This is true, in particular, with regard to the convergence of the power series entering into the formal calculations. This question was left unsettled in the paper cited.

The principal object of the present paper is to complete the equilong theory by showing that the series in question are always convergent. It thus follows that the equality of the absolute invariants is a sufficient as well as a necessary condition for the equivalence of two pairs of curves. The method used is to reduce the question to one in differential equations† and then to apply certain existence theorems, for solutions at a singular point, due to Briot and Bouquet.

1. *Calculation of the Invariants.*

The equilong group of the plane consists of all contact transformations which convert straight lines into straight lines in such a way that the distance δ between the points of contact of any two curves on a common tangent remains in-

* See Conformal Geometry, Proceedings of the Fifth International Congress, Cambridge (1912), vol. 2, pp. 81–87.

† Such a reduction is impossible in the conformal theory. We have instead a functional equation, and in some cases, as Dr. Pfeiffer has recently shown, the formal solution is actually divergent. In the language of the paper cited above, invariant relations of infinite order are required in conformal equivalence, but not in equilong equivalence.