GROUPS GENERATED BY TWO OPERATORS OF THE SAME PRIME ORDER SUCH THAT THE CONJUGATES OF ONE UNDER THE POWERS OF THE OTHER ARE COMMUTATIVE.

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§ 1. *Introduction.*

IN a previous article,* devoted to a study of conjugate operators, attention was called to the fact that a complete set of conjugate operators which are not all relatively commutative is transformed under the group according to a substitution group which is at most *triply* transitive. It is easy to prove that this substitution group is at most *doubly* transitive, and, as the p conjugates of order 2 under the holomorph of the group of the prime order p are transformed under this holomorph according to a doubly transitive group, the substitution group in question can actually be doubly transitive.

The most general definition of a complete set of conjugate operators, or conjugate subgroups, of a group G is that the set is composed of the totality of operators, or subgroups, which correspond under one or more of the possible automorphisms of G. It is, however, usually assumed that such a set is composed of the totality of operators, or subgroups, which correspond under at least one of the *inner* automorphisms of G. Since every possible automorphism of any group is an inner automorphism under its holomorph, this difference in the possible definitions of the expression *complete set of conjugates* does not affect the general theorems relating to such sets.

The theorem mentioned in the first paragraph can be materially extended by the consideration of a complete set of conjugate operators involving an operator which is commutative with at least one of the others without being commutative with all of them. When this condition is satisfied the group of transformations of the set is always *simply* transitive. To prove this let s_1, s_2 represent two commutative operators of

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