described show that each such categorical set of postulates will be also "completely independent."

HARVARD UNIVERSITY.

COMPLETE EXISTENTIAL THEORY OF THE POSTULATES FOR WELL ORDERED SETS.

BY PROFESSOR EDWARD V. HUNTINGTON.

(Read before the American Mathematical Society, December 27, 1916.)

A SYSTEM (K, R), where K is a class of elements $A, B, C, \cdots$ and R is a dyadic relation, is called a *well ordered* system when the following conditions are satisfied:*

(*a*) *the system* (K, R) *is a series; and*

(*b*) *every subsystem of* (K, R) *has a leading element.*†

Now when condition (*b*) is added to the conditions (*a*) which define a series, some of the conditions (*a*) become redundant. After eliminating these redundancies, we find the following *three sets of independent postulates for well ordered systems,* each of these three sets being in fact "completely independent" in the sense of E. H. Moore. (The numbering of the postulates is made to conform with that in the preceding note.)

SET I. (POSTULATES 1, 3, 5.)

Postulate 1. $AA \;.\; = \;.\; 0.$ (Irreflexiveness.)

Postulate 3. $A \neq B \;.\; AB \;.\; BA \;: = \;: 0.$ (Asymmetry for distinct elements.)

Postulate 5. *Every subsystem has at least one leading element.* ("Leadership," or the property of being "supplied with leaders.")

---

* G. Cantor, *Math. Annalen*, vol. 49 (1897), p. 208. A. N. Whitehead and B. Russell, Principia Mathematica, vol. 3 (1913), p. 4.

† Here by a *series* we understand any system (K, R) which satisfies any one of the sets of postulates mentioned in the preceding note. A *subsystem* of (K, R) means any system (K', R') such that K' is a subclass of K, and $R' = R$. (Here K' is called a *subclass* of K if every element of K' belongs to K; that is, a subclass is either a part or the whole.) A *leading element* of a system means any element X having the following property: whenever Y is any other element of the system, then $R(XY)$, or simply XY, will be true. (If a system contains only a single element X, then X is a leading element of that system.)