for $i, j, k = 1, 2, \cdots, r$, are the structural constants of G, we have $c_{ijk} + c_{ikj} = 0$.

F. N. COLE,
Secretary.

COMPLETE EXISTENTIAL THEORY OF THE POSTULATES FOR SERIAL ORDER.

BY PROFESSOR EDWARD V. HUNTINGTON.

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THE purpose of this note is to establish the "complete independence"—in the sense defined by E. H. Moore*—of each of three different sets of postulates for serial order.

The first set of postulates (set A) is new and will be found more convenient for many purposes than either of the other sets. Set B dates back to Vailati, 1892.† Set C (a modification of set B and now widely used) was introduced by the present writer in 1905.‡

The universe of discourse considered in each of these sets is the universe of all systems (K, R), in which K is a class of elements, $A, B, C, \cdots$, and R is a dyadic relation; the notation $R(AB)$, or briefly AB, meaning that the relation R holds

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* E. H. Moore, "Introduction to a form of general analysis," Yale University Press (1910), p. 82. An interesting example of a proof of complete independence is given by R. D. Beetle, "On the complete independence of Schimmack's postulates for the arithmetic mean," *Math. Annalen*, vol. 76 (1915), pp. 444-446. [Compare R. Schimmack, "Der Satz vom arithmetischen Mittel in axiomatischer Begründung," *Math. Annalen*, vol. 68 (1909), pp. 125-132.] For a similar discussion of an *almost* completely independent set of postulates, see L. L. Dines, "Complete existential theory of Sheffer's postulates for Boolean algebras," this BULLETIN, vol. 21 (1915), pp. 183-188. [Compare H. M. Sheffer, "A set of five independent postulates for Boolean algebras, with application to logical constants," *Transactions Amer. Math. Society*, vol. 14 (1913), pp. 481-488.]

† G. Vailati, "Sui principî fondamentali della geometria della retta," *Rivista di Matematica*, vol. 2 (1892), pp. 71-75; B. Russell, Principles of Mathematics, vol. 1 (1903), pp. 203, 218-219.

‡ E. V. Huntington, "The continuum as a type of order," reprinted from the *Annals of Mathematics*, vols. 6 and 7 (1905), especially vol. 6, pp. 157-158; second edition, Harvard University Press, 1917, pp. 10-11, J. W. Young, Fundamental Concepts of Algebra and Geometry (1911), p. 68; A. N. Whitehead and B. Russell, Principia Mathematica, vol. 2 (1912), p. 513. (In the present terminology of Whitehead and Russell, a relation which satisfies postulate 1 is said to be "contained in diversity.").