A NOTE ON DISCONTINUOUS SOLUTIONS IN THE CALCULUS OF VARIATIONS.

BY DR. PAUL R. RIDER.

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IT is the object of this note to give the corner conditions and the forms of the Carathéodory Ω-function for Bliss's form of the simplest problem of the calculus of variations, and for an analogous form of the problem in space.

For the purpose of orientation and of introducing notation, a brief résumé of a part of the theory of discontinuous solutions as treated by Bolza* is given here.

1. If at a point $P_0(t_0)$ of a curve $x = x(t)$, $y = y(t)$ that minimizes or maximizes the definite integral

$$\int_{t_1}^{t_2} F[x(t), y(t), x'(t), y'(t)]dt$$

the curve possesses a corner, the corner conditions

$$F_{x'}\Big|^{t_0-0} = F_{x'}\Big|^{t_0+0}, \quad F_{y'}\Big|^{t_0-0} = F_{y'}\Big|^{t_0+0}$$

must be satisfied.

Let $P_1P_0P_2$ be an extremal (that is, a minimizing or maximizing curve) having a corner at P_0, the corner conditions being satisfied. Suppose that the continuity and other conditions usually imposed in the calculus of variations hold for the arcs P_1P_0, P_0P_2 and for the family of extremals

$$x = \phi(t, a), \quad y = \psi(t, a),$$

which contain the extremal arc $E_0 \equiv P_1P_0$ for $a = a_0$. Designate by τ_0 and $\bar{\tau}_0$ respectively the angles that the positive tangents to the arcs P_1P_0 and P_0P_2 at the point P_0 make with the positive x-axis. Then, if it is desired to find on E_a, a neighboring extremal to E_0, a point $P(t)$ and a direction $\bar{\tau}$ such that τ and τ (τ is the positive direction of E_a at P) satisfy the corner conditions, it is necessary to solve for t and $\bar{\tau}$ the equations

$$(1) \qquad F_{x'} - \bar{F}_{x'} = 0, \quad F_{y'} - \bar{F}_{y'} = 0,$$

* Bolza, Vorlesungen über Variationsrechnung, Chapter 8.