A THEOREM CONCERNING CONTINUOUS CURVES.

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IN this paper I propose to show that every continuous curve has the simple property stated below in Theorem 1. Though my proof is worded for the case of a plane curve, it is clear that with a slight change in phraseology it would apply to a curve in a space of any number of dimensions.

LEMMA. *If* $S_1, S_2, S_3, \cdots$ *is a countable sequence of connected,* bounded point sets such that, for every n,* S_n *contains* S'_{n+1},† *then the set of all points that are common to* $S_1, S_2, S_3, \cdots$ *is closed and connected.*

For a proof of this lemma see my paper "On the foundations of plane analysis situs," *Transactions of the American Mathematical Society,* volume 17 (1916), page 137. Cf. also S. Janiszewski and E. Mazurkiewicz, *Comptes Rendus,* volume 151 (1910), pages 199 and 297 respectively.

THEOREM 1. *Every two points of a continuous curve are the extremities of at least one simple continuous arc that lies entirely on that curve.*

Proof. Suppose A and B are two points belonging to the continuous plane curve C. Hahn has shown‡ that the curve C is connected "im kleinen," i. e., that if P is a point of C, ϵ is a positive number and K is a circle, of radius $1/\epsilon$, with center at P, then there exists, within K and with center at P, another circle $K_{\epsilon P}$ such that if X is a point within $K_{\epsilon P}$, and belonging to C, then X and P lie together in some connected subset of C that lies entirely within K. Let $\overline{K}_{\epsilon P}$ denote the set of all points [Y] belonging to C such that Y and P lie together in some connected subset of C that lies entirely within K. Clearly $\overline{K}_{\epsilon P}$ contains $K_{\epsilon P}$,

* A set of points is said to be *connected* if, however it be divided into two mutually exclusive subsets, one of them contains a limit point of the other one.

† If S is a point set, S' denotes the set of points composed of S together with all its limit points.

‡ Hans Hahn, "Ueber die allgemeinste ebene Punktmenge, die stetiges Bild einer Strecke ist," *Jahresbericht der Deutschen Mathematiker-Vereinigung,* vol. 23 (1914), pp. 318–322.