

when the time for action comes. Even at the present moment we can no longer regard the Society merely as one to forward mathematical research in America; the wider interests of the subject have now to be considered, since we form the one large national organization devoted to research in mathematics which can continue its work unhindered by more pressing claims. In these wider interests our opportunities for service may come: the readiness and ability that we show in a realization and fulfilment of them will be the chief measure of our success in the past and promise for the future.

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A THEOREM ON THE CURVES DESCRIBED BY A SPHERICAL PENDULUM.

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1. IN what follows I shall restrict myself to the case considered by Greenhill,* in which the pendulum bob reaches (but does not go above) the horizontal plane of suspension with a non-vanishing horizontal velocity.†

When this assumption is made, there exists an interesting relation between the curve described by the bob and a certain quartic in space. It is the purpose of this note to establish this simple relation.

2. Let r_0 be the distance of the bob from the vertical axis when its horizontal projection crosses the x -axis; $A + 2\pi$ the horizontal angle between two consecutive positions in the plane of suspension; $4K$ the real period in the Jacobian elliptic functions appearing in the solution; $v = t/2w_1$, where w_1 is the real half-period in the Weierstrassian form, and t the time; a, b, c ($a > b > c$) the real roots of the cubic in the general solution; $\mu = \sqrt{(l^2 + a^2)/(l^2 - a^2)}$, where l is the radius of

* Greenhill, *Les Fonctions elliptiques et leurs Applications*, chap. III. Appell, *Traité de Mécanique rationnelle*, t. 1, p. 494. Tannery et Molik, *Eléments de la Théorie des Fonctions elliptiques*, t. 4, p. 188.

† The statement by L. Lévy, on page 161 of his *Fonctions elliptiques*, that in this case x, y, z are doubly periodic functions of the time is not correct.