ments, we may proceed afresh, since D_p is of the same form as D, but contains only the functions $f_2, f_3, \cdots, f_p$. Reasoning as above, we find that these $p - 1$ functions are linearly dependent, and hence also the p given functions, unless the $(p - 2)$-rowed determinant in the upper right-hand corner of D_p vanishes identically. In the latter case we must continue in the same way, until we finally reach a determinant in the upper right-hand corner of D which is not identically zero. But there must be one such, unless $f_p(t)$ is itself identically zero, in which case the given p functions are linearly dependent. The theorem for analytic functions as stated in Mr. Morse's note therefore follows.

It may be of interest to point out that many—if not all—of the theorems on linear dependence in which wronskians or determinants and matrices constructed like wronskians are involved* have their analogues in corresponding theorems in which appear determinants and matrices resembling the determinant D.

HARVARD UNIVERSITY,
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TRANSLATION SURFACES ASSOCIATED WITH LINE CONGRUENCES.

BY PROFESSOR O. E. GLENN.

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§ 1. *Introduction.*

IN a note published in the BULLETIN in 1914† I established an algorism on a class of surfaces associated with line congruences in 3-space, which result by translation from invariants of plane n-lines. It is the purpose of this paper to apply symbolical methods to the study of some properties of these surfaces.

Two non-homogeneous forms of respective orders m, n, in Plücker's line coordinates p_1, p_2, q_1, q_2, r $[r = (pq)]$, considered together, represent a congruence (m, n). For the sake of symmetry let the variables be changed to the homogeneous system

* Such, for instance, as are given by Bôcher, loc. cit., and by Curtiss, *Math. Annalen*, vol. 65 (1908), pp. 282–298.

† BULLETIN, vol. 20, p. 233.